Charged and electromagnetic fields from relativistic quantum geometry

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Abstract

In the Relativistic Quantum Geometry (RQG) formalism recently introduced, was explored the possibility that the variation of the tensor metric can be done in a Weylian integrable manifold using a geometric displacement, from a Riemannian to a Weylian integrable manifold, described by the dynamics of an auxiliary geometrical scalar field θ, in order that the Einstein tensor (and the Einstein equations) can be represented on a Weyl-like manifold. In this framework we study jointly the dynamics of electromagnetic fields produced by quantum complex vector fields, which describes charges without charges. We demonstrate that complex fields act as a source of tetra-vector fields which describe an extended Maxwell dynamics.

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I. INTRODUCTION

The consequences of non-trivial topology for the laws of physics has been a topic of perennial interest for theoretical physicists[1], with applications to non-trivial spatial topologies[2] like Einstein-Rosen bridges, wormholes, non-orientable spacetimes, and quantum-mechanical entanglements.

Geometrodynamics[3] is a picture of general relativity that study the evolution of the spacetime geometry. The key notion of the Geometrodynamics was the idea of *charge without charge*. The Maxwell field was taken to be source free, and so a non-vanishing charge could only arise from an electric flux lines trapped in the topology of spacetime. With the construction of ungauged supergravity theories it was realised that the Abelian gauge fields in such theories were source-free, and so the charges arising therein were therefore central charges[4] and as consequence satisfied a BPS bound[5] where the embedding of Einstein-Maxwell theory into $N = 2$ supergravity theory was used. The significant advantages of geometrodynamics, usually come at the expense of manifest local Lorentz symmetry[6]. During the 70's and 80's decades a method of quantization was developed in order to deal with some unresolved problems of quantum field theory in curved spacetimes[7].

In a previous work[8] was explored the possibility that the variation of the tensor metric must be done in a Weylian integrable manifold using a geometric displacement, from a Riemannian to a Weylian integrable manifold, described by the dynamics of an auxiliary geometrical scalar field θ, in order that the Einstein tensor (and the Einstein equations) can be represented on a Weyl-like manifold. An important fact is that the Einstein tensor complies with the gauge-invariant transformations studied in a previous work[9]. This method is very useful because can be used to describe, for instance, nonperturbative back-reaction effects during inflation[10]. Furthermore, it was introduced the relativistic quantum dynamics of θ by using the fact that the cosmological constant Λ is a relativistic invariant. In this letter, we extend our study to complex charged fields that act as the source of vector fields A^{μ}.

II. RQG REVISITED

The first variation of the Einstein-Hilbert (EH) action $\mathcal{I}^1$

$$\mathcal{I} = \int_V d^4x \sqrt{-g} \left[\frac{R}{2\kappa} + \mathcal{L}_m \right],\tag{1}$$

is given by

$$\delta\mathcal{I} = \int d^4x\sqrt{-g}\left[\delta g^{\alpha\beta}\left(G_{\alpha\beta} + \kappa T_{\alpha\beta}\right) + g^{\alpha\beta}\delta R_{\alpha\beta}\right],\tag{2}$$

where $\kappa = 8\pi G$, G is the gravitational constant and $g^{\alpha\beta}\delta R_{\alpha\beta} = \nabla_\alpha \delta W^\alpha$, where $\delta W^\alpha = \delta\Gamma^\alpha_{\beta\gamma}g^{\beta\gamma} - \delta\Gamma^\epsilon_{\beta\epsilon}g^{\beta\alpha} = g^{\beta\gamma}\nabla^\alpha\delta\Psi_{\beta\gamma}$. When the flux of δW^α that cross the Gaussian-like hypersurface defined in an arbitrary region of the spacetime, is nonzero, one obtains in the last term of (2), that $\nabla_\alpha\delta W^\alpha = \delta\Phi(x^\alpha)$, such that $\delta\Phi(x^\alpha)$ is an arbitrary scalar field that takes into account the flux of δW^α across the Gaussian-like hypersurface. This flux becomes zero when there are no sources inside this hypersurface. Hence, in order to make $\delta\mathcal{I} = 0$ in (2), we must consider the condition: $G_{\alpha\beta} + \kappa T_{\alpha\beta} = \Lambda g_{\alpha\beta}$, where Λ is the cosmological constant. Additionally, we must require the constriction $\delta g_{\alpha\beta}\Lambda = \delta\Phi g_{\alpha\beta}$. Then, we propose the existence of a tensor field $\delta\Psi_{\alpha\beta}$, such that $\delta R_{\alpha\beta} \equiv \nabla_\beta\delta W_\alpha - \delta\Phi g_{\alpha\beta} \equiv \Box\delta\Psi_{\alpha\beta} - \delta\Phi g_{\alpha\beta} = -\kappa\,\delta S_{\alpha\beta}^{\ 2}$, and hence $\delta W^\alpha = g^{\beta\gamma}\nabla^\alpha\delta\Psi_{\beta\gamma}$, with $\nabla^\alpha\delta\Psi_{\beta\gamma} = \delta\Gamma^\alpha_{\beta\gamma} - \delta^\alpha_\gamma\delta\Gamma^\epsilon_{\beta\epsilon}$. *Notice that the fields $\delta\bar{W}_\alpha$ and $\delta\bar{\Psi}_{\alpha\beta}$ are gauge-invariant under transformations*:

$$\delta\bar{W}_\alpha = \delta W_\alpha - \nabla_\alpha\delta\Phi, \qquad \delta\bar{\Psi}_{\alpha\beta} = \delta\Psi_{\alpha\beta} - \delta\Phi g_{\alpha\beta},\tag{3}$$

where the scalar field $\delta\Phi$ complies $\Box\delta\Phi = 0$. On the other hand, we can make the transformation

$$\bar{G}_{\alpha\beta} = G_{\alpha\beta} - \Lambda g_{\alpha\beta},\tag{4}$$

and the transformed Einstein equations with the equation of motion for the transformed gravitational waves, hold

$$\bar{G}_{\alpha\beta} = -\kappa\,T_{\alpha\beta},\tag{5}$$

$$\Box\delta\bar{\Psi}_{\alpha\beta} = -\kappa\,\delta S_{\alpha\beta},\tag{6}$$

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[1] Here, g is the determinant of the covariant background tensor metric $g_{\mu\nu}$, $R = g^{\mu\nu}R_{\mu\nu}$ is the scalar curvature, $R^\alpha_{\mu\nu\alpha} = R_{\mu\nu}$ is the covariant Ricci tensor and $\mathcal{L}_m$ is an arbitrary Lagrangian density which describes matter. If we deal with an orthogonal base, the curvature tensor will be written in terms of the connections: $R^\alpha_{\ \beta\gamma\delta} = \Gamma^\alpha_{\ \beta\delta,\gamma} - \Gamma^\alpha_{\ \beta\gamma,\delta} + \Gamma^\epsilon_{\ \beta\delta}\Gamma^\alpha_{\ \epsilon\gamma} - \Gamma^\epsilon_{\ \beta\gamma}\Gamma^\alpha_{\ \epsilon\delta}$.

[2] We have introduced the tensor $S_{\alpha\beta} = T_{\alpha\beta} - \frac{1}{2}T g_{\alpha\beta}$, which takes into account matter as a source of the Ricci tensor $R_{\alpha\beta}$.

with $\Box \delta\Phi(x^\alpha) = 0$ and $\delta\Phi(x^\alpha)\, g_{\alpha\beta} = \Lambda\, \delta g_{\alpha\beta}$. The eq. (5) provides us the Einstein equations with cosmological constant included, and (6) describes the exact equation of motion for gravitational waves with an arbitrary source $\delta S_{\alpha\beta}$ on a closed and curved space-time. A very important fact is that the scalar field $\delta\Phi(x^\alpha)$ appears as a scalar flux of the tetra-vector with components δW^α through the closed hypersurface $\partial\mathcal{M}$. This arbitrary hypersurface encloses the manifold by down and must be viewed as a 3D Gaussian-like hipersurface situated in any region of space-time. This scalar flux is a gravitodynamic potential related to the gauge-invariance of δW^α and the gravitational waves $\delta\bar{\Psi}_{\alpha\beta}$. Other important fact is that since $\delta\Phi(x^\alpha)\, g_{\alpha\beta} = \Lambda\, \delta g_{\alpha\beta}$, the existence of the Hubble horizon is related to the existence of the Gaussian-like hypersurface. The variation of the metric tensor must be done in a Weylian integrable manifold[8] using an auxiliary geometrical scalar field θ, in order to the Einstein tensor (and the Einstein equations) can be represented on a Weyl-like manifold, in agreement with the gauge-invariant transformations (3). If we consider a zero covariant derivative of the metric tensor in the Riemannian manifold (we denote with ";" the Riemannian-covariant derivative): $\Delta g_{\alpha\beta} = g_{\alpha\beta;\gamma}\, dx^\gamma = 0$, hence the Weylian covariant derivative $g_{\alpha\beta|\gamma} = \theta_\gamma\, g_{\alpha\beta}$, described with respect to the Weylian connections [3]

$$\Gamma^\alpha_{\beta\gamma} = \left\{ \begin{array}{c} \alpha \\ \beta\ \gamma \end{array} \right\} + g_{\beta\gamma}\theta^\alpha, \tag{7}$$

will be nonzero

$$\delta g_{\alpha\beta} = g_{\alpha\beta|\gamma}\, dx^\gamma = -\left[\theta_\beta g_{\alpha\gamma} + \theta_\alpha g_{\beta\gamma} \right]\, dx^\gamma. \tag{8}$$

A. Gauge-invariance and quantum dynamics

From the action's point of view, the scalar field $\theta(x^\alpha)$ is a generic geometrical transformation that leads invariant the action

$$\mathcal{I} = \int d^4x\, \sqrt{-\hat{g}} \left[\frac{\hat{R}}{2\kappa} + \hat{\mathcal{L}} \right] = \int d^4x \left[\sqrt{-\hat{g}}e^{-2\theta} \right] \left\{ \left[\frac{\hat{R}}{2\kappa} + \hat{\mathcal{L}} \right] e^{2\theta} \right\}, \tag{9}$$

where we shall denote with a hat, $\hat{\ }$, the quantities represented on the Riemannian manifold. Hence, Weylian quantities will be varied over these quantities in a Riemannian manifold so

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[3] To simplify the notation we shall denote $\theta_\alpha \equiv \theta_{,\alpha}$

that the dynamics of the system preserves the action: $\delta\mathcal{I} = 0$, and we obtain

$$-\frac{\delta V}{V} = \frac{\delta\left[\frac{\hat{R}}{2\kappa} + \hat{\mathcal{L}}\right]}{\left[\frac{\hat{R}}{2\kappa} + \hat{\mathcal{L}}\right]} = 2\,\delta\theta, \tag{10}$$

where $\delta\theta = -\theta_\mu dx^\mu$ is an exact differential and $V = \sqrt{-\hat{g}}$ is the volume of the Riemannian manifold. Of course, all the variations are in the Weylian geometrical representation, and assure us gauge invariance because $\delta\mathcal{I} = 0$. Using the fact that the tetra-length is given by $S = \frac{1}{2}x_\nu\hat{U}^\nu$ and the Weylian velocities are given by $u^\mu = \hat{U}^\mu + \theta^\mu\left(x_\epsilon\hat{U}^\epsilon\right)$, can be demonstrated that

$$u^\mu u_\mu = 1 + 4S\left(\theta_\mu\hat{U}^\mu - \frac{4}{3}\Lambda\,S\right). \tag{11}$$

The components u^μ are the relativistic quantum velocities, given by the geodesic equations

$$\frac{du^\mu}{dS} + \Gamma^\mu_{\alpha\beta}u^\alpha u^\beta = 0, \tag{12}$$

such that the Weylian connections $\Gamma^\mu_{\alpha\beta}$ are described by (23). In other words, the quantum velocities u^μ are transported with parallelism on the Weylian manifold, meanwhile $\hat{U}^\mu$ are transported with parallelism on the Riemann manifold. If we require that $u^\mu u_\mu = 1$, we obtain the gauge

$$\hat{\nabla}_\mu A^\mu = \frac{2}{3}\Lambda^2\,S(x^\mu). \tag{13}$$

Hence, we obtain the important result

$$d\Phi = \frac{1}{6}\Lambda^2\,S\,dS, \tag{14}$$

or, after integrating

$$\Phi(x^\mu) = \frac{\Lambda^2}{12}\,S^2(x^\mu), \tag{15}$$

such that $d\Phi(x^\mu) = -\frac{\Lambda}{2}d\theta(x^\mu)$. Hence, from eq. (9) we obtain that the quantum volume is given by

$$V_q = \sqrt{-\hat{g}}\,e^{-2\theta} = \sqrt{-\hat{g}}\,e^{\frac{1}{3}\Lambda S^2}, \tag{16}$$

where $\Lambda S^2 > 0$. This means that $V_q \geq \sqrt{-\hat{g}}$, for $S^2 \geq 0$, $\Lambda > 0$ and $\theta < 0$. This implies a signature for the metric: $(-,+,+,+)$ in order for the cosmological constant to be positive and a signature $(+,-,-,-)$ in order to have $\Lambda \leq 0$. Finally, the action (9) can be rewritten in terms of both, quantum volume and the quantum Lagrangian density $\mathcal{L}_q = \left[\frac{\hat{R}}{2\kappa} + \hat{\mathcal{L}}\right]e^{2\theta}$

$$\mathcal{I} = \int d^4x\,V_q\,\mathcal{L}_q. \tag{17}$$

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As was demonstrated in [8] the Einstein tensor can be written as

$$\bar{G}_{\alpha\beta} = \hat{G}_{\mu\nu} + \theta_{\alpha;\beta} + \theta_\alpha\theta_\beta + \frac{1}{2}\, g_{\alpha\beta}\left[(\theta^\mu)_{;\mu} + \theta_\mu\theta^\mu\right], \tag{18}$$

and we can obtain the invariant cosmological constant Λ

$$\Lambda = -\frac{3}{4}\left[\theta_\alpha\theta^\alpha + \hat{\Box}\theta\right], \tag{19}$$

so that we can define a geometrical Weylian quantum action $\mathcal{W} = \int d^4x\,\sqrt{-\hat{g}}\,\Lambda$, such that the dynamics of the geometrical field, after imposing $\delta W = 0$, is described by the Euler-Lagrange equations which take the form

$$\hat{\nabla}_\alpha\Pi^\alpha = 0, \qquad \text{or} \qquad \hat{\Box}\theta = 0, \tag{20}$$

where the momentum components are $\Pi^\alpha \equiv -\frac{3}{4}\theta^\alpha$ and the relativistic quantum algebra is given by[8]

$$[\theta(x),\theta^\alpha(y)] = -i\Theta^\alpha\,\delta^{(4)}(x-y), \qquad [\theta(x),\theta_\alpha(y)] = i\Theta_\alpha\,\delta^{(4)}(x-y), \tag{21}$$

with $\Theta^\alpha = i\hbar\,\hat{U}^\alpha$ and $\Theta^2 = \Theta_\alpha\Theta^\alpha = \hbar^2\hat{U}_\alpha\,\hat{U}^\alpha$ for the Riemannian components of velocities $\hat{U}^\alpha$.

B.   Charged geometry and vector field dynamics

In order to extend the previous study we shall consider that the scalar field θ is given by

$$\theta(x^\alpha) = \phi(x^\alpha)\,e^{-i\theta(x^\alpha)}, \qquad \text{or} \qquad \theta(x^\alpha) = \phi^*(x^\alpha)\,e^{i\theta(x^\alpha)}, \tag{22}$$

where $\phi(x^\alpha)$ is a complex field and $\phi^*(x^\alpha)$ its complex conjugate. In this case, since $\theta^\alpha = e^{i\theta}\left(\hat{\nabla}^\alpha + i\theta^\alpha\right)\phi^*$, the Weylian connections hold

$$\Gamma^\alpha_{\beta\gamma} = \left\{{\alpha \atop \beta\,\gamma}\right\} + e^{i\theta}\,g_{\beta\gamma}\left(\hat{\nabla}^\alpha + i\,\theta^\alpha\right)\phi^* \equiv \left\{{\alpha \atop \beta\,\gamma}\right\} + g_{\beta\gamma}\,e^{i\theta}\left(D^\alpha\phi^*\right), \tag{23}$$

where we use the notation $D^\alpha\phi^* \equiv \left(\hat{\nabla}^\alpha + i\theta^\alpha\right)\phi^*$. The Weylian components of the velocity u^μ and the Riemannian ones U^μ, are related by

$$u^\mu = \hat{U}^\mu + e^{i\theta}\left(D^\mu\phi^*\right)\left(x_\epsilon\hat{U}^\epsilon\right). \tag{24}$$

Furthermore, using the fact that

$$\delta g_{\alpha\beta} = e^{-i\theta} \left[\left(\hat{\nabla}_\beta - i\theta_\beta \right) \hat{U}_\alpha + \left(\hat{\nabla}_\alpha - i\theta_\alpha \right) \hat{U}_\beta \right] \phi \, \delta S, \tag{25}$$

we can obtain from the constriction $\Lambda \delta g_{\alpha\beta} = g_{\alpha\beta} \delta \Phi$, that

$$\delta \Phi = \frac{\Lambda}{4} g^{\alpha\beta} \, \delta g_{\alpha\beta}, \tag{26}$$

so that, using (25), the flux of A^μ across the Gaussian-like hypersurface can be expressed in terms of the quantum derivative of the complex field:

$$\frac{\delta \Phi}{\delta S} \equiv \frac{d\Phi}{dS} = \frac{\Lambda}{2} e^{i\theta} \hat{U}_\alpha \left(D^\alpha \phi^* \right). \tag{27}$$

Using the fact that $\hat{\nabla}_\alpha \delta W^\alpha = \delta \Phi$, it is easy to obtain

$$\hat{\nabla}_\mu A^\mu = \frac{\Lambda}{2} e^{i\theta} \hat{U}_\alpha \left(D^\alpha \phi^* \right), \tag{28}$$

where we have defined $A^\mu = \frac{\delta W^\mu}{\delta S}$. Notice that the velocity components $\hat{U}^\alpha$ of the Riemannian observer define the gauge of the system. Furthermore, due to the fact that $\delta W^\alpha = g^{\beta\gamma} \hat{\nabla}^\alpha \delta \Psi_{\beta\gamma}$, hence we obtain that

$$\frac{\delta W^\alpha}{\delta S} \equiv A^\alpha = g^{\beta\gamma} \hat{\nabla}^\alpha \chi_{\beta\gamma} \equiv \hat{\nabla}^\alpha \chi, \tag{29}$$

where $\chi_{\beta\gamma}$ are the components of the gravitational waves:

$$\hat{\nabla}_\alpha A^\alpha = g^{\beta\gamma} \hat{\nabla}_\alpha \hat{\nabla}^\alpha \chi_{\beta\gamma} \equiv \hat{\Box} \chi. \tag{30}$$

III. QUANTUM FIELD DYNAMICS

In this section we shall study the dynamics of charged and vector fields, in order to obtain their dynamical equations.

A. Dynamics of the complex fields

The cosmological constant (19) can be rewritten in terms of $\phi = \theta \, e^{i\theta}$ and $\phi^* = \theta e^{-i\theta}$

$$\Lambda = -\frac{3}{4} \left[\left(\hat{\nabla}_\nu \phi \right) \left(\hat{\nabla}^\nu \phi^* \right) + \theta_\nu \, J^\nu - \frac{4}{3} \Lambda \phi \phi^* \right], \tag{31}$$

where the current due to the charged fields is

$$J^\nu = i \left[\delta^\nu_\epsilon \left(\hat{\nabla}^\epsilon \phi \right) \phi^* - \phi \left(\hat{\nabla}^\nu \phi^* \right) \right]. \tag{32}$$

As can be demonstrated, $\hat{\nabla}_\nu J^\nu = 0$, so that we obtain the condition

$$\phi^* \, e^{i \left(\theta - \frac{\pi}{2} \right)} = \phi \, e^{-i \left(\theta - \frac{\pi}{2} \right)}. \tag{33}$$

The components of the current also can be written in terms of the quantum derivative

$$J^\mu = -2 \left(1 + i \right) \phi \, e^{i\theta} \left(D^\mu \phi^* \right) \phi, \tag{34}$$

where the density of electric charge is given by J^0, and the charge is

$$Q = -2 \left(1 + i \right) \int d^3 x \sqrt{|\det[g_{ij}]|} \, \phi(x^\alpha) \, e^{i\theta} \left[D^\mu \phi^*(x^\alpha) \right] \phi(x^\alpha). \tag{35}$$

The second equation in (20) results in two different equations

$$\left(\hat{\Box} + i\theta_\mu \hat{\nabla}^\mu + \frac{4}{3} \Lambda \right) \phi^* = 0, \tag{36}$$

$$\left(\hat{\Box} - i\theta^\mu \hat{\nabla}_\mu + \frac{4}{3} \Lambda \right) \phi = 0, \tag{37}$$

where the gauge equations are

$$- \left[i\theta_\mu \hat{\nabla}^\mu + \frac{3}{4} \Lambda \right] \phi^* = \frac{3}{4} \Lambda \, e^{-i \left(\theta - \frac{\pi}{2} \right)}, \tag{38}$$

$$\left[i\theta^\mu \hat{\nabla}_\mu - \frac{3}{4} \Lambda \right] \phi = \frac{3}{4} \Lambda \, e^{i \left(\theta - \frac{\pi}{2} \right)}, \tag{39}$$

so that finally we obtain the equations of motion for both fields

$$\hat{\Box} \phi^* = \frac{3}{4} \Lambda \, e^{-i \left(\theta - \frac{\pi}{2} \right)}, \tag{40}$$

$$\hat{\Box} \phi = \frac{3}{4} \Lambda \, e^{i \left(\theta - \frac{\pi}{2} \right)}. \tag{41}$$

Notice that the functions $e^{\pm i \left(\theta - \frac{\pi}{2} \right)}$ are invariant under $\theta = 2 \, n\pi$ (n- integer) rotations, so that the complex fields are vector fields of spin 1. Using the expressions (23) to find the commutators for the complex fields, we obtain that

$$[\phi^*(x), D^\mu \phi^*(y)] = \frac{4}{3} i \Theta^\mu \, \delta^{(4)}(x - y), \qquad [\phi(x), D_\mu \phi(y)] = -\frac{4}{3} i \Theta_\mu \, \delta^{(4)}(x - y), \tag{42}$$

where $D^\mu \phi^* \equiv \left(\hat{\nabla}^\mu + i \, \theta^\mu \right) \phi^*$ and $D_\mu \phi \equiv \left(\hat{\nabla}_\mu - i \, \theta_\mu \right) \phi$.

B.   Dynamics of the vector fields

On the other hand, if we define $F^{\mu\nu} \equiv \hat{\nabla}^\mu A^\nu - \hat{\nabla}^\nu A^\mu$, such that A^α is given by (29), we obtain the equations of motion for the components of the electromagnetic potentials A^ν: $\hat{\nabla}_\mu F^{\mu\nu} = J^\nu$

$$\hat{\Box} A^\nu - \hat{\nabla}^\nu \left(\hat{\nabla}_\mu A^\mu \right) = J^\nu, \tag{43}$$

where J^ν being given by the expression (32) and from eq. (13) we obtain that $\hat{\nabla}_\mu A^\mu = -\frac{\Lambda}{2}\theta_\mu \hat{U}^\mu = \frac{2}{3}\Lambda^2 S(x^\mu) = 4\frac{d\Phi}{dS}$ determines the gauge that depends on the Riemannian frame adopted by the relativistic observer. Notice that for massless particles the Lorentz gauge is fulfilled, but it does not work for massive particles, where $S \neq 0$.

IV.   FINAL REMARKS

We have studied charged and electromagnetic fields from relativistic quantum geometry. In this formalism the Einstein tensor complies with gauge-invariant transformations studied in a previous work[9]. The quantum dynamics of the fields is described on a Weylian manifold which comes from a geometric extension of the Riemannian manifold, on which is defined the classical geometrical background. The connection that describes the Weylian manifold is given in eq. (23) in terms of the quantum derivative of the complex vector field with a Lagrangian density described by the cosmological constant (31). We have demonstrated that vector fields A^μ describe an extended Maxwell dynamics [see eq. (43)], where the source is provided by the charged fields current density J^μ, with tetra-divergence null. Furthermore, the gauge of A^μ is determined by the relativistic observer: $\hat{\nabla}_\mu A^\mu = \frac{\Lambda}{2}\theta_\mu \hat{U}^\mu$.

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[1] H. Weyl, *Philosophy of Mathematics and Natural Science*, english version, Princeton University Press (1949).

[2] M. Cvetic, G. W. Gibbons, C. N. Pope, **JHEP 1503**: (2015) 029.

[3] J. A. Wheeler, Annals Phys. **2**: 604 (1957);

J. A. Wheeler, *Superspace and the Nature of Quantum Geometrodynamics*, in Battelle Rencontres, 1967 Lectures in Mathematics and Physics, edited by C. M. De Witt and J. A. Wheeler (W. A. Benjamin, New York, 1968).

[4] G. W. Gibbons. Lect. Notes Phys. **160**: (1982) 145.

[5] G. W. Gibbons, C. M. Hull, Phys. Lett. **B109**: (1982) 190.

[6] I. Rácz, Class. Quant. Grav. **32** (2015) 015006.

[7] E. Prugovečki, Ann. Phys. (N.y.) **110**: (1978) 102;

E. Prugovečki, Phys. Rev. **D18**: (1978) 3655;

E. Prugovečki, Nuo. Cim. **A97**: (1987) 597.

[8] L. S. Ridao, M. Bellini, Phys. Lett. **B751**: (2015) 565.

[9] L. S. Ridao, M. Bellini, Astrophys. Space Sci., **357** (2015) 1, 94.

[10] M. Bellini, Phys. Dark Univ. **11**: 64 (2016).